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Comparison of Meteorological Data with Fitted Values Extracted from a Trajectory

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Atmospheric conditions are found by knowing only the projectile's trajectory and its aerodynamic coefficients plus initial atmospheric conditions at ground level. The governing equations are the modified point-mass equations, which are used to generate a test trajectory using data taken with a weather balloon. A nonlinear least-square method is used to fit the modified point-mass equations to the test trajectory by varying the meteorological parameters. Linear wind profiles now replace the previous constant wind profiles, and the yaw of repose is now included in the fitting procedure. A more complete error analysis is presented. Density, temperature, and wind profiles agree well.

Nomenclature

A = $[\rho S\ell/2m]C_D V/\ell$
AZ = azimuth of the 1 axis, measured clockwise from north, rad
B = $k_a^2(\rho S\ell/2m)(V/\ell)^2$, 1/s²
C_D = drag coefficient, where $|\text{drag force}| = (\rho V^2 S/2)C_d$
C_{L_p} = roll damping moment coefficient, where $|\text{roll damping moment}| = (\rho V^2 S\ell/2)(\dot\phi\ell/V)C_{L_p}$
C_{L_α} = gradient of lift force coefficient, where $|\text{lift force}| = (\rho V^2 S/2)|\alpha_e|C_{L_\alpha}$
C_{M_α} = gradient of pitching moment coefficient, where $|\text{static moment}| = \pm(\rho V^2 S\ell/2)|\alpha_e|C_{M_\alpha}$
$C_{N_{p\alpha}}$ = gradient of Magnus force coefficient, where $|\text{Magnus force}| = \pm(\rho V^2 S/2)(\dot\phi\ell/V)|\alpha_e|C_{N_{p\alpha}}$
C_1, C_2, C_3, C_4 = parameters to fit the wind, m/s
D = $h_a(\boldsymbol{G}\cdot\boldsymbol{V})/(1+h_a)V^2$, 1/s
E = gun elevation, rad
$\boldsymbol{G}$ = gravity plus Coriolis acceleration, $\boldsymbol{g} - 2\boldsymbol{\omega}\times\boldsymbol{U}$
$\boldsymbol{G}_A$ = $1/(1+h_a)\{\boldsymbol{G} + [h_L(\boldsymbol{G}\times\boldsymbol{V})/(1-h_M)V]\}$, m/s²
g = $|\boldsymbol{g}|$, m/s²
$\boldsymbol{g}$ = gravity acceleration, m/s²
g_0 = $|\boldsymbol{g}_0|$, 9.80665 m/s²
H = geopotential altitude, m
H_A = value of H at the starting time of a fitting interval, m
H_B = value of H at the bottom of an altitude zone, m
h_a = $[h_L^2/(1-h_M)] - h_M$
h_L = $k_a^2(C_{L_\alpha}/C_{M_\alpha})(\dot\phi\ell/V)$
h_M = $k_a^2(C_{N_{p\alpha}}/C_{M_\alpha})(\dot\phi\ell/V)^2$
I_x = axial moment of inertia, kg m²
k_a^2 = $I_x/m\ell^2$
L = latitude at the launch point; for southern hemisphere firings, replace L by $-L$
ℓ = reference length equal to diameter, m
M = molecular weight, kg/mol
m = projectile mass, kg
Q = $(g_0 M)/\Re$
R = effective radius of the Earth; 6,356,766 m
$\Re$ = universal gas constant, J/mol K
S = reference area, $\pi\ell^2/4$, m
T = temperature, K
T' = temperature gradient dT/dH, K/km
T_B = temperature at altitude H_B, K
t = time, s
$\boldsymbol{U}$ = projectile velocity (U_1, U_2, U_3) with respect to the Earth, m/s
$\boldsymbol{V}$ = projectile velocity (V_1, V_2, V_3) with respect to air, m/s
$\boldsymbol{W}$ = wind velocity (W_1, W_2, W_3) with respect to the Earth, m/s
$\boldsymbol{X}$ = projectile position (X_1, X_2, X_3) with respect to the Earth, m
x = X_1 distance downrange
y = X_2 distance vertical
y_ε = $dy/d\varepsilon$
Z = x/ℓ length (caliber)
z = altitude, m
α = proportionality error parameter $\delta_x = \alpha\delta_y$
α_e = yaw of repose, rad
δ = total positional error $\sqrt{(\delta_x^2 + \delta_y^2)}/\ell$
$\boldsymbol{\delta}_x$ = positional error x direction
$\boldsymbol{\delta}_y$ = positional error y direction
ε = perturbation parameter
ρ = air density, kg/m³
ρ_B = air density at H_B, kg/m³
$\dot\phi$ = axial spin rate, rad/s
$\boldsymbol{\omega}$ = angular velocity of the Earth, rad/s
$(')$ = $d(\ )/dx$
$(\dot{\ })$ = $d(\ )/dt$

Introduction

ARTILLERY accuracy is strongly dependent on current knowledge of weather conditions; the more recent the information the better the accuracy.[1] Weather balloons are used to gather the temperature, pressure, and lateral wind velocity as functions of altitude above the Earth's surface. The data are presented in a formalized meteorological table format referred to as a met message. Continually changing weather causes the information to become stale, which contributes significantly to the artillery round's error budget. Ideally, atmospheric conditions are needed at the time a round is fired.[1]

Because the trajectory of a fired projectile strongly depends on the atmospheric conditions, the problem can be inverted to determine the atmospheric conditions when given this trajectory. Hence, the atmospheric conditions would be determined soon after firing the projectile such that the elapsed time for the met message depends on the computer's speed and the efficiency of the solution algorithm. Atmospheric quantities could subsequently be used to point the artillery piece to a new azimuth.

Cooper et al.[2] have obtained solutions to the inverse problem, that is, finding the met message that influences a given trajectory. Using a FORTRAN subroutine for fitting nonlinear equations (FINLIE),[3] they fit a simple parameterized atmospheric model to successive trajectory segments that cover the given trajectory from ground to just

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below apogee. Because of a lack of adequate trajectory measurements, the given input trajectory was calculated from the modified point-mass (MPM) equations[3] incorporating measured atmospheric data from a weather balloon.

Reported here is a continuation of the work of Cooper et al.[2] The previous fitted lateral wind velocity profiles were assumed constant, over each trajectory segment, which are now replaced by linear profiles. We also have now included the yaw of repose in the sum-of-squares cost function.[3] A more comprehensive error analysis predicts that the necessary segment lengths, needed to maintain a specified parameter error tolerance, have the same order of magnitude as those found previously.[2]

Trajectory Equations and Coordinate System

The MPM equations are assumed to be the governing equations of motion[2]

$$\dot{\boldsymbol{U}} = (D - A)\dot{\boldsymbol{V}} + [1/(1 + h_a)]\{\boldsymbol{G} + [h_L/(1 - h_M)V]\boldsymbol{G} \times \boldsymbol{V}\}$$

$$\ddot{\phi} = -BC_{L_p}(\dot{\phi}\ell/V) \tag{1}$$

where

$$\boldsymbol{\omega} = \omega_E[\cos(L)\cos(AZ), \sin(L), -\cos(L)\sin(AZ)]\cdots$$

$$\omega_E \approx 7.291 \times 10^{-5}\ 1/\mathrm{s}$$

$$C_D = C_{D_0} + C_{D_2}|\boldsymbol{\alpha_e}|^2$$

where

$$\boldsymbol{\alpha}_e = \dot{\phi}\boldsymbol{G} \times \boldsymbol{V}/BV^2C_{M\alpha}$$

called yaw of respose. $C_{L\alpha}$, $C_{M\alpha}$, C_{D_0}, and C_{L_p} are tabulated functions of Mach number and $(C_{D_2}, C_{N_{p\alpha}})$ are constant.

As before,[2] we use a coordinate system with launch point at the origin of a right-handed set of Cartesian axes located at sea level. The 1 axis and 3 axis form a plane tangent to the Earth at the origin, and the 2 axis is perpendicular to this plane with positive pointing up. At launch time $t = 0$ the 1 axis is coincident with the initial velocity vector $\boldsymbol{U}_0$, $\ni U_0 = |\boldsymbol{U}_0|[\cos(E), \sin(E), 0]$. For our purposes the gravity vector can be written as $\boldsymbol{g} = g_0\boldsymbol{X} \times \boldsymbol{Re}_2/|\boldsymbol{X} \times \boldsymbol{Re}_2|$. Formerly, the influence equations, used to construct the cost function needed by FINLIE,[3] did not include the yaw of repose. This inclusion will be shown to have a significant contribution to the fitting analysis when the projectile is near apogee.

Atmospheric Input Data

Because a measured trajectory is not readily available, we continue to calculate a test input trajectory. This is generated using balloon data taken at Aberdeen Proving Ground (APG) on 30 March 1992 in the early afternoon. The data consist of pressure, temperature, and horizontal wind velocities initially taken at very small intervals of altitude gradually increasing to intervals of 400 m for altitudes above 3000 m.

Atmosphere Model Used with the Fitting Equations

Cooper et al.[2] fit constant horizontal components of the wind velocity, as well as density and temperature, in a multiple-segmented atmosphere. We now extend this to linear winds such that the atmosphere is modeled by

$$W_1 = C_1 + C_2(H - H_B), \qquad T = T_B + T'(H - H_B)$$

$$W_2 = 0, \qquad \rho/\rho_B = (T/T_B)^{-1-Q/T'}$$

$$W_3 = C_3 + C_4(H - H_B)$$

$$Q = g_0M/\Re = 34.16319474\ \mathrm{deg\ K/km}$$

$$H = z/[z/(R + 1)] = \text{the geopotential altitude}$$

$$\text{for altitude } z = \sqrt{(R + X_2)^2 + X_1^2} - R \tag{2}$$

The subscript B denotes the values at the lower boundary of an atmospheric segment, and $(C_{j'}s, T')$ are the fitting constants found by FINLIE.

The projectile considered here is the M107 (Ref. 4) projectile, which is well studied, so that all coefficients needed to integrate the MPM equations are well known. This projectile has the following physical properties: ℓ(diameter) = 155 mm, m (mass) = 43.09 kg m, $I_x = 0.1462$ kg m^2, and $(C_{D_2}, C_{N_{p\alpha}}) = (4.0, -0.75)$. The muzzle velocity is 700 m/s, and the initial spin rate is 1443/s. The same aerodynamic behavior is assumed for both the fitting equations and the generated trajectory equations. It is generally conceded that the drag coefficient is known to within a half percent. The significance of this will now be explored.

Preliminary Error Analysis

To gain insight into the way that measurement errors can propagate, we consider a more complete analysis than what was given before.[2] The limiting form of the MPM equations where drag and gravity are the only acting forces are

$$\ddot{X}_1 = -\varepsilon v\rho(X_2)\dot{X}_1, \qquad \ddot{X}_2 = -\varepsilon v\rho(X_2)\dot{X}_2 - g_0$$

$$v = \sqrt{\dot{X}_1^2 + \dot{X}_2^2}, \qquad \varepsilon = C_DS/2m$$

For convenience let $X_1 = x$, $X_2 = y$, $(\cdot) = \dot{x}(')$, and then $y'' = -g_0/\dot{x}^2$ and $(\dot{x}^2)' = -2\varepsilon\rho(y)v\dot{x}$. Hence,

$$y''' = 2\varepsilon\rho(y)y''\sqrt{(y')^2 + 1}, \qquad \ni y(0) = 0, \qquad y|_{x=0} = \tan(L)$$

$$y''|_{x=0} = -g_0/V_0^2\cos(L)^2$$

where L is the launch angle with respect to the horizontal and V_0 is the initial velocity. Assuming $y(x)$ to be a power series in ε along with taking $\rho(y)$ to be given by Eq. (2) with $z \approx y$, one will find that $y(x)$ is well approximated for $V_0 = 700$ m/s and $L = \pi/4$ by

$$\begin{aligned} y(x) = &-\left(\frac{2\varepsilon^3}{265{,}165} - \frac{\varepsilon^2}{1{,}046{,}306{,}816} + \frac{\varepsilon}{81{,}099{,}854{,}184{,}448}\right)x^5 \\ &- \left(\frac{1{,}048{,}577\varepsilon^2}{78{,}643{,}242{,}232} - \frac{\varepsilon}{1{,}880{,}584{,}192}\right)x^4 \\ &- \left(\frac{116\varepsilon}{6{,}151{,}829}\right)x^3 - \left(\frac{1}{50{,}000}\right)x^2 + x \end{aligned} \tag{3}$$

Denote the measurement errors in x, y, and (air density and drag) as δ_x, δ_y, and δ_ε. Assuming the proportionality $(\delta_x = \alpha\delta_y)$ and considering the usual probable error relation $\delta_y^2 = \delta_x^2(y')^2 + \delta_\varepsilon^2(y_\varepsilon)^2$ gives

$$\delta^2 = -\frac{(\alpha^2 + 1)\delta_\varepsilon^2(y_\varepsilon/L)^2}{\alpha^2(y')^2 - 1} \qquad \text{for} \qquad \delta^2 = \frac{\delta_x^2 + \delta_y^2}{\ell^2} \tag{4}$$

Equations (3) and (4) can now be used to calculate the distance $x = Z\ell$ as a function of δ and $\varepsilon/\varepsilon_0 \ni \varepsilon_0 = 9.4 \times 10^{-5}$ for typical errors in drag and density measurements given by $\delta_\varepsilon/\varepsilon = 0.01$. The plots shown in Figs. 1 and 2 are sample calculations for the M107 projectile fired at 700 m/s with $L = \pi/4$ elevation such that $\alpha = 1$ giving predictions for the new analysis [Eqs. (3) and (4)] and the old analysis.[2] These results indicate that the required segment lengths are on the same order of magnitude as given by the earlier predictions. However, this more general analysis considers a segment starting at ground level with altitude dependent density $\rho(y)$. An uncertainty in segment length on the order of 1 m ($\delta = 1/0.155$ m) can still require a large altitude length, ≈1700 m, when $\varepsilon/\varepsilon_0 = 0.2$, which might preclude obtaining sufficiently accurate meteorological estimates in the altitude zones. This analysis can be extended to any altitude segment starting at y_b by substituting $y(x) \rightarrow y(x) - y_b$ in the preceding equations with the appropriate boundary conditions at y_b.

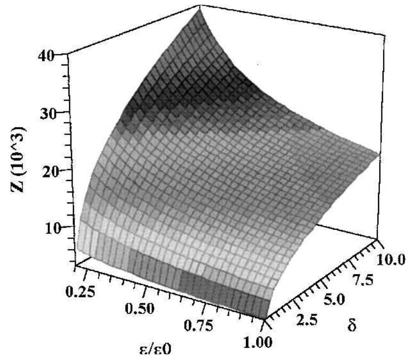

Fig. 1 Required segment length to achieve given accuracy: new length estimate.

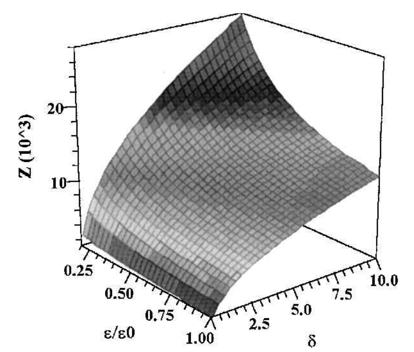

Fig. 2 Required segment length to achieve given accuracy: old length estimate.

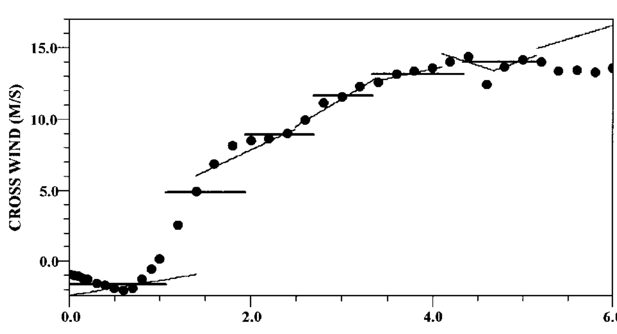

Fig. 3 Comparison of crosswind velocity W_3 data with fitted values: ●, measured; ——, new results; and ▬, old results.

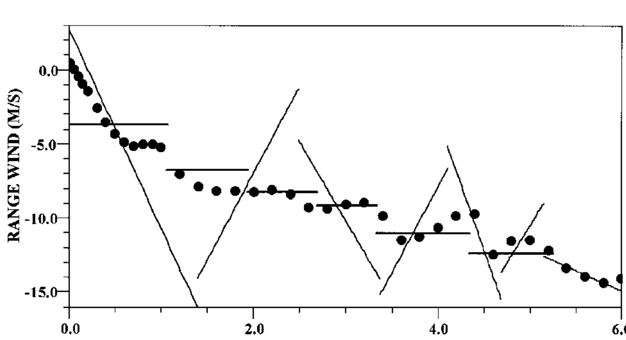

Fig. 4 Comparison of range wind velocity W_1 data with fitted values: ●, measured; ——, new results; and ▬, old results.

Results

The extended met model was fitted in the same sequential fashion as before[2] starting at ground level and using parameter values at the end of one trajectory segment for initial guesses in the next segment. Measured wind velocity data along with the fitted curves, resulting from the least-squares fitting method, are shown in Figs. 3 and 4. These profiles show an overall improvement with linear fits replacing the preceding constant functions. The linear functions plus the inclusion of the yaw of repose have now made it possible to extend the fitting procedure all the way to apogee, which previously was found, in general, not to be possible because of a lack of convergence.

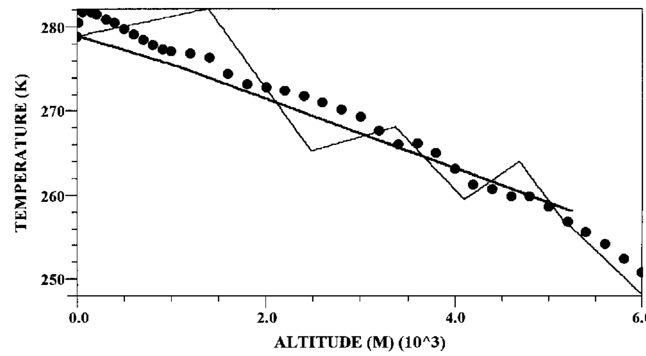

Fig. 5 Comparison of temperature data with fitted values: ●, measured; ——, new results; and ▬, old results.

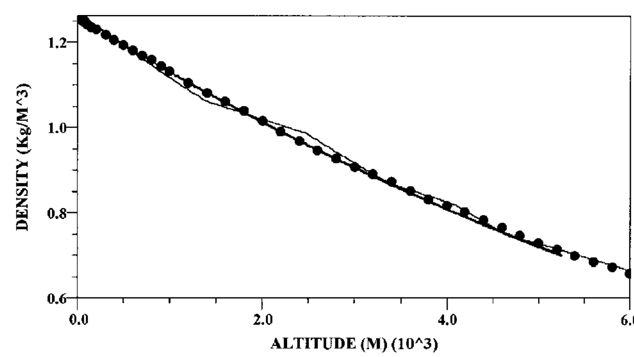

Fig. 6 Comparison of density with fitted values: ●, measured; ——, new results; and ▬, old results.

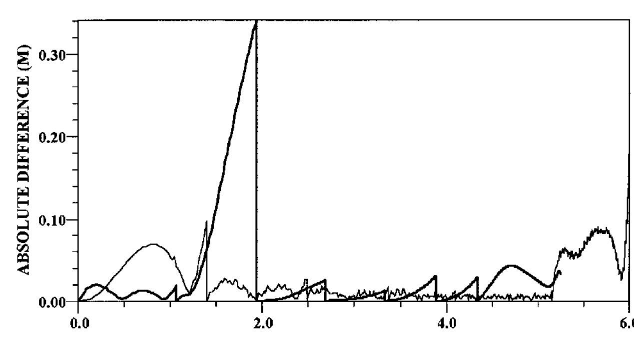

Fig. 7 Error in fitted position (meters) vs altitude: ——, new calculated errors, and ▬, old calculated errors.

Density and temperature data are compared with both new and old fitted values in Figs. 5 and 6. These plots show that both the new fitted temperature and density bracket their respective measured values from ground to apogee. A pointwise comparison shows that both the temperature and density fits have degraded, but in the next section one will see that a pointwise comparison of the projectile's positional differences has improved.

Error Results

Temperature and therefore density are important influences on a projectile's trajectory. Temperature errors chiefly affect the coefficient C_{D_0}, which depends weakly on temperature for supersonic speeds, more strongly for transonic speeds, and very little for subsonic speeds. Winds are the next important influence on the trajectory. Hence, one way to quantify results is to compare the absolute difference between the input trajectory and a trajectory produced by a converged fitted met. Figure 7 displays the differences as functions of altitude ranging from ground level to apogee for the new procedure and just below apogee as limited by the old procedure. This plot shows that the extended atmosphere model considerably reduces the maximum error. The area under the old curve is 169.18 m^3, whereas the area under the new curve is 143.13 m^3, which further shows that the linear wind model provides better results than those found using the constant wind model.

Conclusions

Atmospheric conditions are calculated knowing only the trajectory and the projectiles aerodynamic coefficients. The input trajectory was generated from the MPM equations using meteorological data taken with a weather balloon. To solve the inverse problem, that is, to get the met message that produced a given trajectory, we minimize a cost function,[3] which is essentially the sum-of-the-square differences between the input and the fitted trajectories, using a least-squares fitting method. The agreement with measured balloon velocity data has improved when compared to the previous results.[2] However, the temperature and density agree less than what was found earlier[2] even though the trajectory fit has significantly improved. Including the yaw of repose, the cost function has also allowed these fitted curves to range from ground to apogee as was previously unattainable.

References

[1] Matts, J. A., and Ellis, A. G., "Artillery Accuracy: Simple Models to Assess the Impact of New Equipment and Tactics," U.S. Army Ballistic Research Lab., Rept. BRL-TR- 3101, Aberdeen Proving Ground, MD, April 1990.

[2] Cooper, G., Fansler, K., and Oskay, V., "Extracting Meteorological Data from Projectile Trajectory," *Journal of Spacecraft and Rockets*, Vol. 34, No. 6, 1997, pp. 769–773.

[3] Bradley, J. W., "FINLIE: A FORTRAN Program for Fitting Ordinary Differential Equations with Nonlinear Parameters to Data," U.S. Army Ballistic Research Lab., Rept. BRL-TR-02290 (AD A098038), Aberdeen Proving Ground, MD, Feb. 1981.

[4] MacAllister, L. C., and Krial, K. S., "Aerodynamic Properties and Stability Behavior of the 155 mm Howitzer Shell, M107," U.S. Army Ballistic Research Lab., Rept. BRL-MR-2547, Aberdeen Proving Ground, MD, Oct. 1975.

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